Title: Rigid Geometry and Motivic Zeta Functions

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Let X be a connected smooth complex variety, and consider a dominant morphism f from X to the affine line Spec C[t]. Denef and Loeser associated to f a motivic zeta function Z(f;T), by analogy with Igusa's local zeta function in the p-adic setting. This motivic zeta function is a generating series with coefficients in a certain Grothendieck ring, and it reflects the arithmetic properties of X over C[[t]]. The monodromy conjecture predicts a precise connection between the poles of Z(f;T), and certain invariants of the singularities of the complex hypersurface X(0) defined by f (namely, the monodromy eigenvalues).

The t-adic completion of f is a formal scheme over C[[t]], and we call its generic fiber the nearby fiber of f. It is a smooth quasi-compact rigid variety over the field of Laurent series C((t)) (rigid geometry is a theory of analytic geometry over non-archimedean fields, such as C((t)) endowed with its t-adic norm). We will show how formal and rigid geometry can be applied to the study of the zeta function Z(f;T). In particular, we establish this function as a Weil generating series, measuring rational points on the nearby fiber over finite extensions of C((t)), and we prove a Grothendieck trace formula. This yields an interesting connection between the arithmetic properties of the nearby fiber, and the geometry of the singularities of f.

We introduce the analytic Milnor fiber of f at a closed point x of X(0) (this is an open rigid subvariety of the nearby fiber of f), and we show that it determines the singularity of f at x completely. Its étale cohomology, with Galois action, corresponds to the singular cohomology of the topological Milnor fiber of f at x, endowed with the monodromy action; its points are closely related to the coefficients of the motivic zeta function.