

Title: Smallest poles of Igusa's p -adic zeta functions

Speaker: Dirk Segers (University of Leuven, Belgium)

Igusa's p -adic zeta function $Z_f(s)$ of a polynomial $f \in \mathbb{Z}[x_1, \dots, x_n]$ is related to the Poincaré series of f , and thus also to its coefficients M_i , which count the number of solutions of the polynomial congruence $f \equiv 0 \pmod{p^i}$. A lot of people have already studied this zeta function. In particular, the study of its poles is fascinating because they determine the asymptotic behaviour of the M_i and because they are related to the monodromy conjecture.

On the one hand, we obtain for $n = 2$ and $n = 3$ results about the smallest real part of the poles of $Z_f(s)$ by using an embedded resolution of f . We need a new formula for the residue of $Z_f(s)$ at a candidate pole of expected order one to be able to prove that some candidate poles are not poles. As a consequence, we get nice properties for the M_i .

On the other hand, we obtain for arbitrary n a result for the M_i by using only elementary techniques. In this way we get a lower bound for the real part of the poles of $Z_f(s)$ without using the technique of embedded resolution of singularities. Our bound is optimal.